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Subsystem identification in Statistical Energy Analysis

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TREBALL FINAL DE GRAU

Abstract

One of the most important things when we want to solve a mechanical or vibroacoustic problem falls on subdivision of the study domain, which directly affects the process. Thus, in this thesis we present a detailed study of the subsystem identification technique, that in the case of SEA (Statistical Energy Analysis) method is based on the classification of the domain according to transfer energy between subsystems.

A subsystem is a set of nodes with a similar behaviour, which allows us to subdivide a domain into different groups. This subdivision is achieved by performing a cluster analysis of the transfer matrix powers. The Transfer matrix, which we obtain from the mechanical properties of our engineering structure, allows us to describe and quantify the direct transmission of signals between nodes.

The identification of subsystems in complex or large geometries is very useful in the SEA method applied to vibroacoustic problems, but also to the reduction of the operational cost in a finite element problems, such as it has been done in this thesis, optimising the Newmark method.

All these concepts are implemented, a posteriori, in the analysis of vibroacoustics problems through different representative geometries, in order to analyse the accuracy and applicability of the method.

Keywords: Subsystem, coupling, transfer matrix, operational cost, cluster analysis.

Resum

Una de les parts més importants a l'hora de voler resoldre un sistema mecànic o vibroacústic recau en la subdivisió del domini d'estudi, afectant directament en el procés. Així doncs, en la present tesi es realitza un estudi detallat de la tècnica d'identificació de subsistemes, que en el cas del mètode SEA (Statistical Energy Analysis) es basa en la classificació del domini en funció de l'energia de transferència entre els subsistemes.

Un subsistema és un conjunt de nodes amb un comportament similar, que ens permet subdividir un domini en diferents grups. Aquesta subdivisió s'aconsegueix realitzant una anàlisi clúster de les potències de la matriu de transferència. La matriu de transferència, obtinguda arran de les propietats mecàniques de la nostra estructura, ens permet descriure i quantificar la transmissió directa de senyal entre nodes.

Aquesta identificació de subsistemes en geometries complexes o de grans dimensions, és de gran utilitat en l'aplicació del mètode SEA a problemes vibroacústics, però també ho és per a la reducció del cost operacional d'un problema d'elements finits, tal com hem realitzat en aquesta tesi, optimitzant el mètode de Newmark.

Tots aquests conceptes són posats en pràctica a posteriori a l'anàlisi de problemes vibroacústics mitjançant diferents geometries representatives, amb el fi d'analitzar la precisió i aplicabilitat del mètode.

Paraules clau: subsistema, acoblament, matriu de transferència, cost operacional, anàlisi clúster.

Agraïments

Aquesta tesi de final de grau d'Enginyeria Civil de l'Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports de Barcelona és un treball en el qual han participat directament i indirectament diverses personalitats, institucions i companys, que m'han permès millorar com a estudiant i com a persona.

Primerament, he de realçar i agrair el gran esforç, paciència i ajuda prestada pels tutors i col·laboradors del meu TFG; Antonio Rodríguez Ferran, Jordi Poblet Puig i Francesc Xavier Magrans Fontrodona, que m'han permès, entre d'altres, estar al costat d'un projecte d'investigació del LaCàN (Laboratori de Càlcul Numèric).

També he d'agrair la gran predisposició del CIMNE (International Center for Numerical Methods in Engineering) i en particular, dos dels seus investigadors: Poyan Dadvan i Ricarrdo Rossi, que m'han ajudat a entendre molt millor Kratos i poder aprofitar els codis amagats del programa. En especial he de recalcar la gran ajuda donada per Ricarrdo Rosi, que també em va permetre obtenir una llicència anual de GiD.

Des del punt de vista teòric de les estructures, donar les gràcies al professor Luis Miguel Cervera Ruiz, per estar sempre disposat a resoldre qualsevol dubte.

Finalment, agrair als companys: Pere Arrom Coll i Ivan Cantero Sánchez, per les seves opinions, punts de vista i ànims que han fet que aquest treball fossi molt més enriquidor.

Contents

List of Figures

List of Tables

1 Preamble

1.1 Introduction

Numerical analysis is a mathematical branch that has been booming as they have been appearing and improving computers. Nowadays, in the field of engineering, they are practically used for everything, because their speed to obtain solutions to complex problems and even to problems that analytically do not have expression, makes up for reduction of accuracy.

In this project we focus particularly on tools that are applied in the SEA method but could be exported to other methodologies, which are based on the detailed analysis of large data packets.

The analysis of large volumes of data, from the more conventional point of view (solving the integrally system of equations), entails a very high operational cost, which affects the time it takes to solve the problem, and ultimately; to the economic cost of a project. Thus, finding methods that allow us to reduce these costs without losing much precision, are very important for any company.

That is why in this thesis we will focus on the identification of subsystems, as this allows us to carry out detailed studies of specific areas of our problem, without having to calculate and analyse the whole, reducing the times. But not only that, because if we divide a problem into several subsystems, we can have several computers working in parallel, sharing the necessary information between them.

Therefore, we will see this characteristics applied to the vibroacoustic problem, but can be extended to other fields, not only in structures, but to the general analysis of data.

1.2 Objectives

The main objectives of the thesis are:

- To understand satisfactorily the method and all its theoretical base, so that we can apply it efficiently in practical problems and at the same time to evaluate its precision and application.
- Proceed to check the automation process of the method, analysing all the main tools.
- Observe how the method affects operational cost through representative examples.

From a more generic and personal point of view, my main objectives are as follows:

- Implement numerical processes in a realistic way, improving my knowledge of Structures and Numerical Methods.
- Improve my knowledge of programming using Matlab.
- Understand and improve my Kratos domain applied to structures.

2 Subsystems identification technique

The following sections will explain all the necessary concepts to understand the subsystem identification technique.

2.1 Statistical Energy Analysis for vibroacoustics

In the vibroacoustic problems it is common to use the SEA (Statistical Energy Analysis) method that it is based on the study of the transmission of acoustic or vibroacoustic signal in a range of high frequencies. This method uses the paths analysis to identify subsystems with the same energy behaviour, isolating weakly coupled systems for to be able to study them in detail reducing significantly the operational cost.

A subsystem is a set of elements with a similar behaviour, which in our case are the nodes of the finite element mesh. In particular for the SEA method, it is studied that the behaviour has an energetic similarity and not geometric or mechanical similarity. Sometimes these subsystems coincide, but many other times do not.

Therefore, paths theory analyses how the signal is transmitted between the nodes of a mechanical system. When this signal is large it is said that the subsystems are strongly coupled and when the signal is weak they are weakly coupled subsystems.

The main way to quantify and describe analytically this behaviour is the transfer matrix and its powers. In the case of to study directly the transfer matrix, we will describe the first-order paths, and when we operate with their powers, the K-order paths will be described.

2.2 Transfer matrix

As explained in the previous section, the transfer matrix (T) shows the relationship between subsystems and it is obtained from the following process:

$$Ax = b \qquad with \quad x_i, b_i, a_{ij} \in \mathbb{C} \tag{1}$$

Also,

$$A = D + L + U \tag{2}$$

Where "D" is the main diagonal of "A", "L" is the strictly upper triangular matrix and U is the strictly lower triangular matrix, and "A" is invertible matrix.

With Eq.(1) and Eq.(2) we obtain:

$$x = D^{-1}b + Tx \tag{3}$$

Finally,

$$T = -D^{-1}(L + U) \tag{4}$$

To obtain "D", "L" and "U", now "A" is defined in relation to our engineering structure as:

$$A = K - \omega^2 M \tag{5}$$

Where "K" is the stiffness matrix, "M" the mass matrix and "ω" the study frequency.

More information can be found in the reference [1] and [2].

2.3 Stiffness matrix

In order to obtain the stiffness matrix, the GiD software has been used, in particular Kratos, developed by the International Center for Numerical Methods in Engineering (CIMNE). This program has been chosen because it is simple to generate any geometry, as complicated as we want, and obtain its matrix of stiffness, connectivity and the position matrix of the nodes in a simple way. For that purpose, we had to modify some parameters from the source files of the program, because in the first instance we could only obtain the connectivity and position matrix.

2.4 Mass matrix

The mass matrix can be defined as Consistent or Concentrated form. In the case that concerns us and since we are going to work with inverse matrices, we will use the Concentrated form because we can assure the existence of inverse matrix and its calculation is direct.

The Concentrated mass matrix is generated much more simply than the Consistent form. This divides the total mass of our object by components (in our case nodes), so for each component of our geometry has assigned the nth part of the total mass.

$$M = \frac{\rho V}{n}[Id] \tag{6}$$

M = Mass matrix [N].

ρ = density [kg/m^3].

V = volume [m^3].

n = number of nodes.

$[Id]$ = identity matrix.

In our particular case, we need to have the mass matrix by degrees of freedom and not by nodes, which is why we proceed to study in detail what happens in a dense system of nodes, since the degrees of freedom in displacements will have a different character from the degrees of freedom in rotations, therefore, can not have the same percentage of mass.

With the following mesh chosen for our study, Fig. 1, we will see what happens with degrees of freedom.

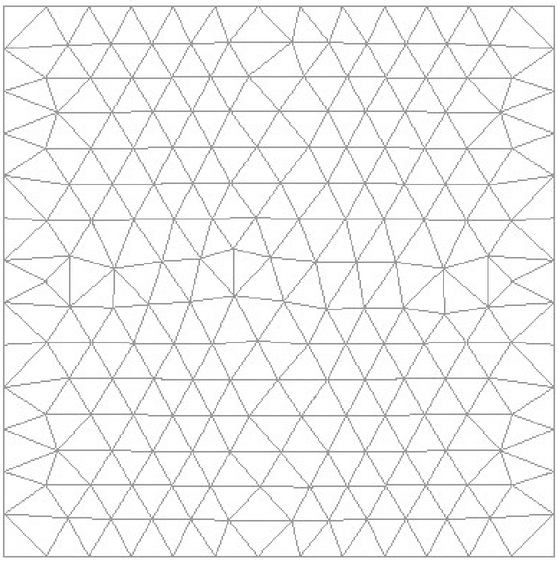


Figure 1: General mesh.

If we produce a unitary displacement in the "y" direction of the node "i" (we do not lose generality by choosing the displacement "y") and in parallel we produce a unitary rotation in the "z" direction (conjugate rotation to the displacement "y"), we can clearly see that there are different results, Fig. 2 and Fig. 3 respectively. In dense mesh the nodes will be very close, and for this we will study the end case.

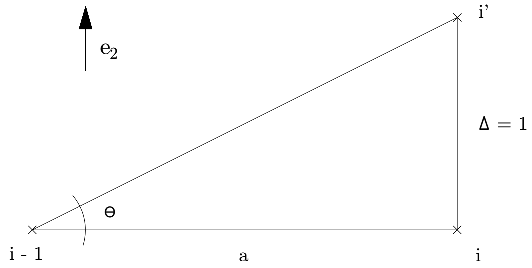


Figure 2: Displacements.

$$tan(\theta) = \frac{\Delta}{a} = \frac{1}{a} \tag{7}$$

$$\lim_{a \to 0} \frac{1}{a} = \infty \tag{8}$$

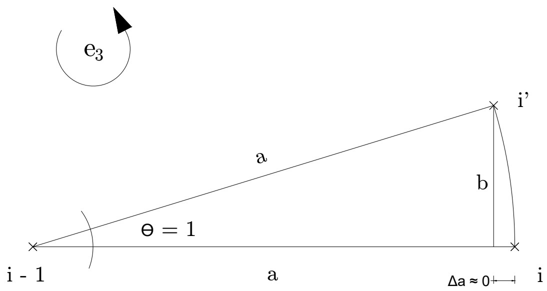


Figure 3: Rotations.

$$b = a \sin(\theta) = a \sin(1) \tag{9}$$

$$\lim_{a \to 0} a \sin(1) = 0 \tag{10}$$

As we can see, in very close nodes, the result of forcing a unitary rotation is much smaller than forcing a unitary displacement. The unitary displacement produces an angle of 90 °, whereas a unitary rotation produces a zero displacement. Therefore, the mass percentage for degrees of freedom in rotations can be assumed to be null, this is a good simplification for the calculations. It has to be emphasized that the current software such as SAP2000 or Kratos assume this hypothesis in their structures algorithms.

3 Tools to automate the technique

Once the subsystem identification technique is understood, now our goal is to find an automatic way to apply it. In the following sections we will be able to analyse different tools that we will implement in the representative examples. More information can be found in the reference [3].

3.1 Colour plot of the Transfer matrix

One of the most important parts of a result is that it is easily understood for anyone. Thus, we use the "colourmap" function of Matlab that allows us to observe by colours, according to a scaled range, the different values and their relation. In other words, the extreme values of the matrix have extreme colours of the scale, as we can see in the following figures, Fig. 4 and Fig. 5.

$$
\begin{matrix}
5 & 5 & 5 & 5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 5 & 5 & 5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 5 & 5 & 5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 5 & 5 & 5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 5 & 5 & 5 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 & 5 & 5 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 \\
0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 & 5 & 5 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 \\
0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 & 5 & 5 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 \\
0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 & 5 & 5 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 \\
0 & 0 & 0 & 0 & 0 & 5 & 5 & 5 & 5 & 5 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 \\
0 & 0 & 0 & 0 & 0 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 & 5 & 5 & 5 & 5 & 5 \\
0 & 0 & 0 & 0 & 0 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 & 5 & 5 & 5 & 5 & 5 \\
0 & 0 & 0 & 0 & 0 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 & 5 & 5 & 5 & 5 & 5 \\
0 & 0 & 0 & 0 & 0 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 & 5 & 5 & 5 & 5 & 5 \\
0 & 0 & 0 & 0 & 0 & 2.5 & 2.5 & 2.5 & 2.5 & 2.5 & 5 & 5 & 5 & 5 & 5
\end{matrix}
$$

Figure 4: Example matrix.

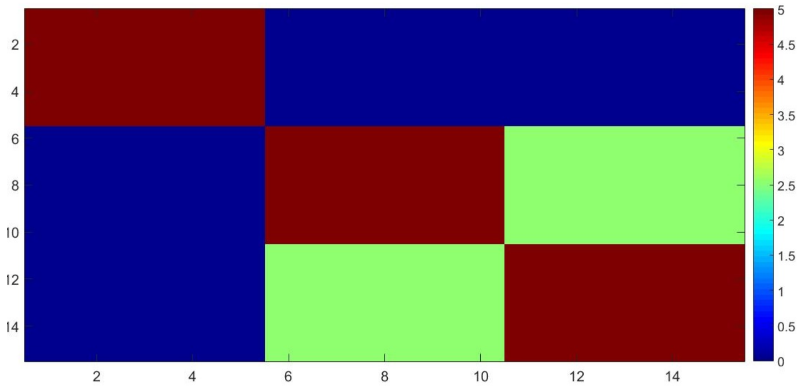


Figure 5: General colour plot.

We will use this tool to see if the transfer matrix is defined by blocks or it is all uniform without any pattern.

Moreover, this tool will be useful for different situations:

a) In case that the transfer matrix is weakly defined by blocks, it means that the matrix has not converged yet and we have to increase its power.

b) If it has converged to block matrix, it would be useful to see if we can increase the accuracy and delimit better the blocks.

c) In case it does not converge to any solution by blocks, we would indicate that the geometry chosen is strongly coupled.

But all this process is manual, so we need another tool that allows us to do it in a simple, visual and automatic way. This tool is cluster analysis.

3.2 Cluster Analysis

In order to automate all the previous process, we use the cluster analysis.

Cluster analysis use clustering algorithms with the goal of finding patterns or groupings in a dataset. This analysis form groupings or clusters in such a way that data within a cluster have a higher measure of similarity than data in any other cluster. The measure of similarity on which the clusters are modelled can be defined by Euclidean distance, probabilistic distance, or another metric.

3.3 Dendrogram

As in the case of the transfer matrix and the colour plot, the tool that we use now to represent clearly and visually the results of the cluster analysis is the Dendrogram. This allows us to represent groups of cluster analysis in a hierarchical way. With this function we can cut and choose the groups that we want according to our interests. The principal difference between colour plot and Dendrogram is that in addition to obtaining a graphical response, we also get a matrix with the different groups and the elements that within this groups. This is so useful because we can use this information for other sections of our project.

In order to explain it clearly, we will continue with an easy example, Fig. 6:

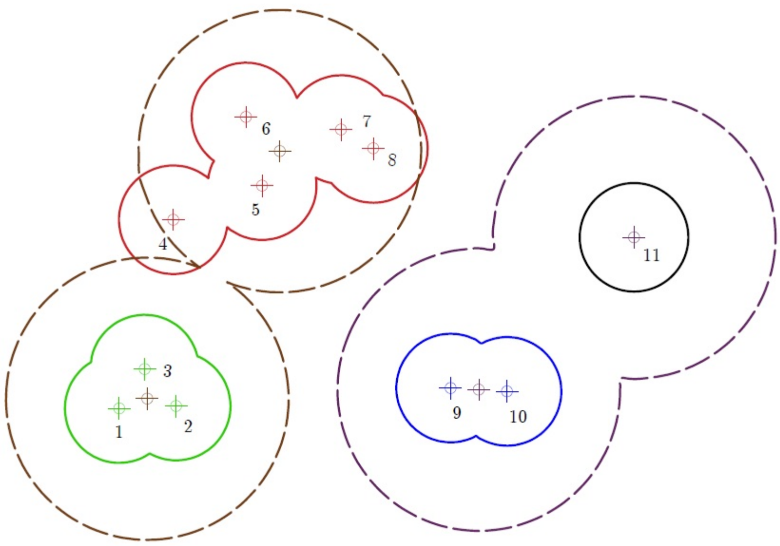


Figure 6: Position nodes of simple example.

In the previous image we can see different points that are grouped by proximity between them generating groups until obtain two final groups. In the following figure, Fig. 7, we can see the same thing but in a tree form.

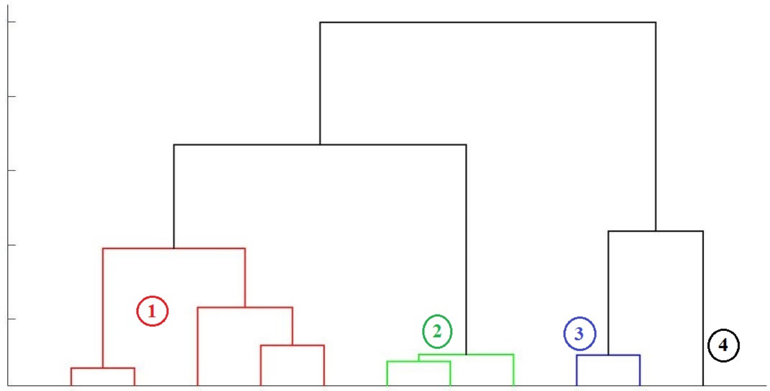


Figure 7: Dendrogram of simple example.

Finally, we can observe that if we make a matrix of distances between the different points and apply the colour plot, seen in the previous section, we get the same result as with the Dendrogram, see Fig. 8. (In this case we will not discuss if the different blocks are strongly or weakly coupled, only we want to identify the different subgroups and groups.)

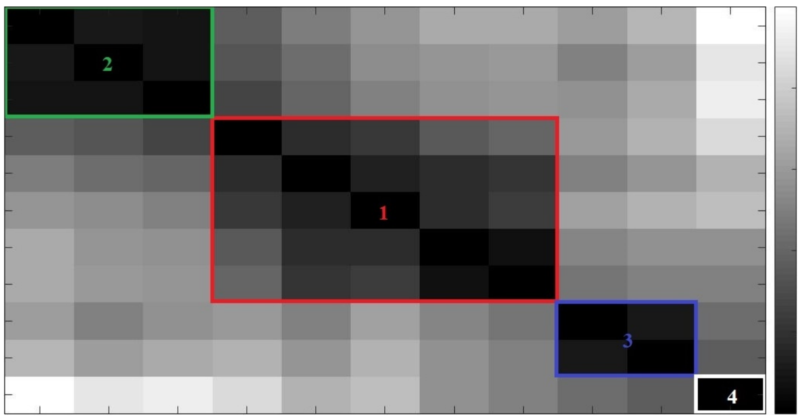


Figure 8: Colour plot of simple example.

3.4 Degree of coupling between subsystems

The degree of coupling of subsystems is the dependence to one another, for the same stress. It can happen that for a stress these subsystems are strongly coupled, and for another stress they are weakly coupled

A quick and automatic way to visualize if the subsystems are strongly coupled is the contrast.

3.4.1 Contrast

Contrast is a tool that compares the inputs of the different subsystems, by means of the transfer matrix or its powers.

$$\begin{bmatrix} T_1 & \alpha T_{12} \\ \alpha T_{21} & T_2 \end{bmatrix} \qquad with \quad \alpha \in (0,1) \tag{11}$$

Thus, analytically, the contrast is defined as:

$$contrast_{12} = \frac{1}{||{T_1}^{-1} \alpha T_{12}||} \tag{12}$$

$$contrast_{21} = \frac{1}{||{T_2}^{-1} \alpha T_{21}||} \tag{13}$$

As we can see in the examples of the next section, the contrast is a simple scalar allows us to know quickly and automatically if they are strongly coupled or not. The greater the contrast, the greater degree of coupling between subsystems.

Finally, in order to be able to assume that the subsystems are uncoupled, the following condition must be met:

$$\frac{||x_1 - {x_1}^1||}{||x_1||} \leq \frac{1}{contrast_{12}} \tag{14}$$

where "x_1" is the fully coupled solution and "${x_1}^1$" the solution of the uncoupled system.

4 Dynamic problem

Our dynamic problem is defined by the following equation, Eq. 15 that relates the stiffness and mass matrices:

$$M\ddot{u} + K\dot{u} = f_{ext} \tag{15}$$

We are in front of a system of partial differential equations (PDE) that can be attacked by means of different methodologies. A good way to solve our problem and easily programmable is the Newmark method.

4.1 Newmark method

The Newmark method is a system widely used in structural dynamics, which is based on a bi-parametric generalization of Taylor's development. The relationships used are:

$$u^{n+1} = u^n + \Delta t\dot{u}^n + \Delta t^2((\frac{1}{2} - \beta)\ddot{u}^n + \beta\ddot{u}^{n+1}) \tag{16}$$

$$\dot{u}^{n+1} = \dot{u}^n + \Delta t((1-\gamma)\ddot{u}^n + \gamma\ddot{u}^{n+1}) \tag{17}$$

If we apply the previous relations to our problem, we will obtain a system of linear equations that we can solve in a conventional way.

$$M\ddot{u}^{n+1} + K\dot{u}^{n+1} = f_{ext}^{n+1} \tag{18}$$

To ensure the stability and convergence of the method, we have to take the following values for the Newmark parameters, the trapezoidal rule:

$$\beta = \frac{1}{4} \quad and \quad \gamma = \frac{1}{2} \tag{19}$$

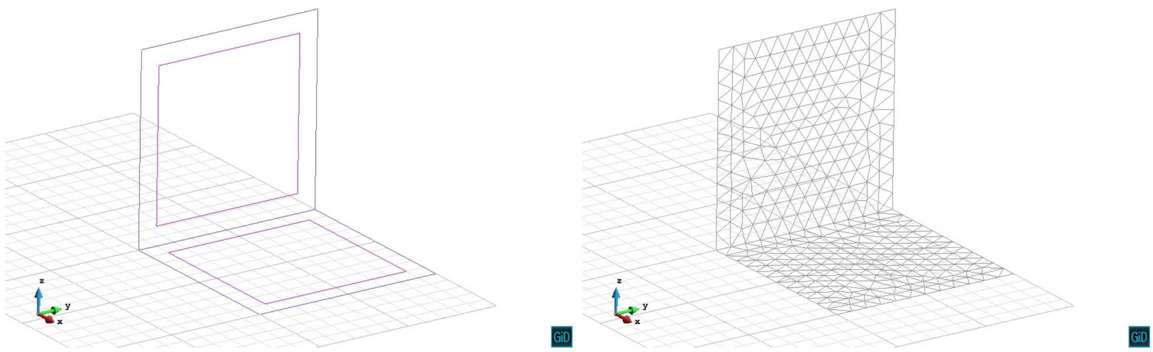
More information can be found in the reference [6].

5 Representative Examples

In order to apply the studied method, we are going to make a series of examples with different properties. We will describe the different materials used and their mechanical properties. Kratos has been used for this step of the project, because it will provide the stiffness matrix, among others, in a simple way as we explained earlier.

5.1 Example 1: Plates in 90º

In the next figures, Fig. 9a and Fig. 9b, we can see the first geometry designed, the union of two plates of steel in 90º, an "L", and the mesh that we chosen for to obtain the different matrices. In Table 1 we find its mechanical properties.



(a) Geometry. (b) Mesh.

Figure 9: Example 1, Plates in L.

Element	Density(kg/m^3)	Poisson Ratio	Young Modulus(Pa)	Thickness(m)
Plates	7850	0.25	206.9·10^9	0.025

Table 1: Mechanical properties of the Example 1.

After choosing the geometry, we proceed to generate the transfer matrix using our Matlab code.

Through the colour plot, after iterating the powers of the transfer matrix, we can see in the first instance that our geometry does not converge to a blocks matrix, see Fig. 10.

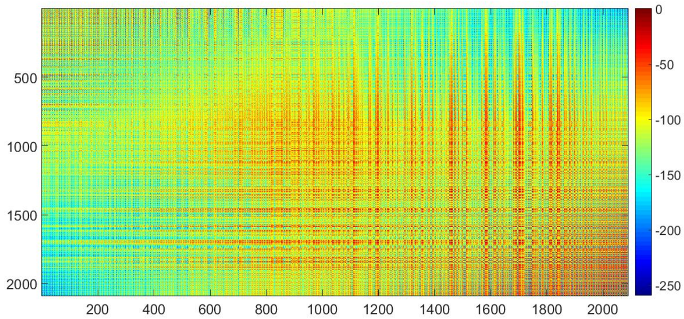


Figure 10: Colour plot of Example 1.

If we do the cluster analysis and the Dendrogram of the results, we can see clearly, that has not been an appropriate identification of subsystems, see the Dendrogram in Fig. 11, and the subgroups in Fig. 12 and Fig. 13.

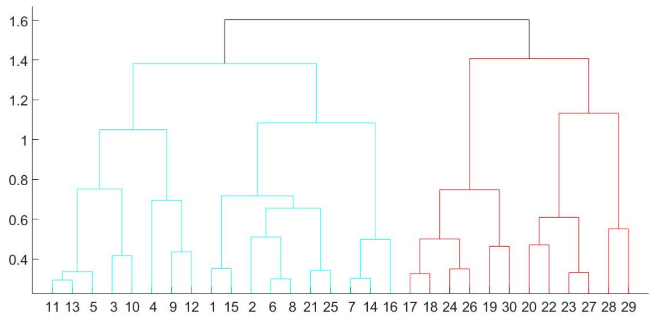


Figure 11: Dendrogram of Example 1.

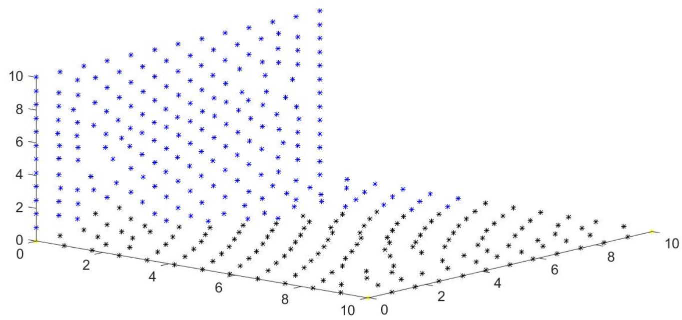


Figure 12: Group 1 of the cluster analysis EX1.

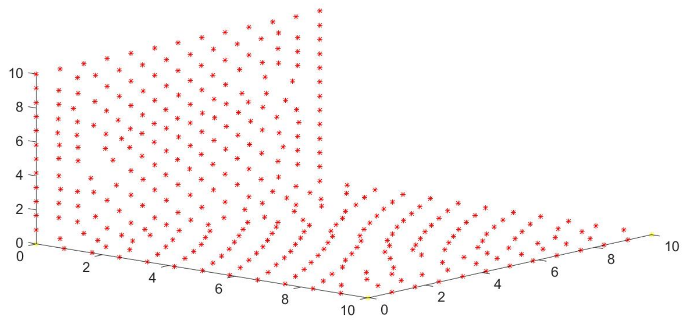


Figure 13: Group 2 of the cluster analysis EX1.

As a consequence, we will try to improve the identification of subsystems. Now let's just analyse the degrees of freedom in displacements, because the displacements and rotations have different units and this could be distorting the result.

We proceed in the same way as in the previous case. We will see that the transfer matrix will not converge to blocks matrix, see Fig. 14. At the same time, the analysis cluster can not identify the subgroups, see Fig. 15, Fig. 16 and Fig. 17.

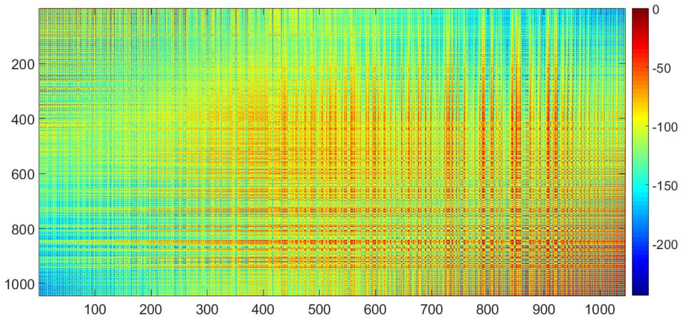


Figure 14: Colour plot of Example 1, displacements.

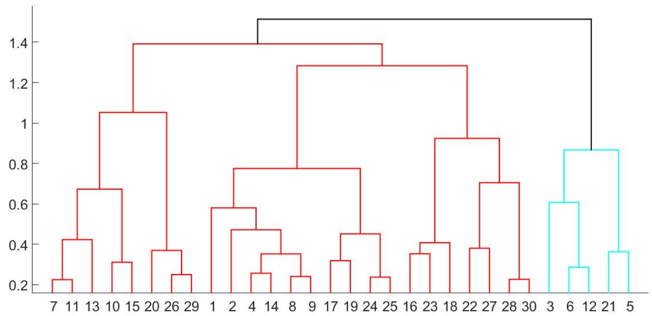


Figure 15: Dendrogram of Example 1, displacements.

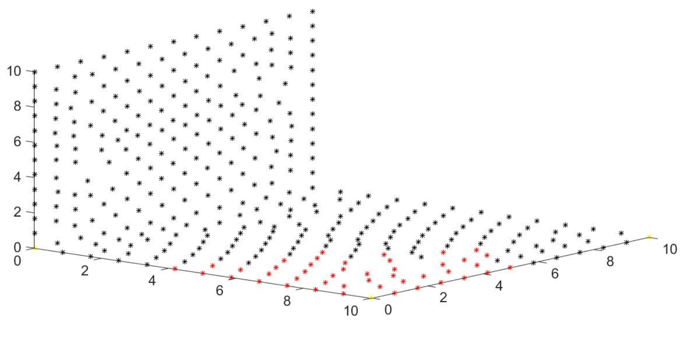


Figure 16: Group 1 of the cluster analysis EX1, displacements.

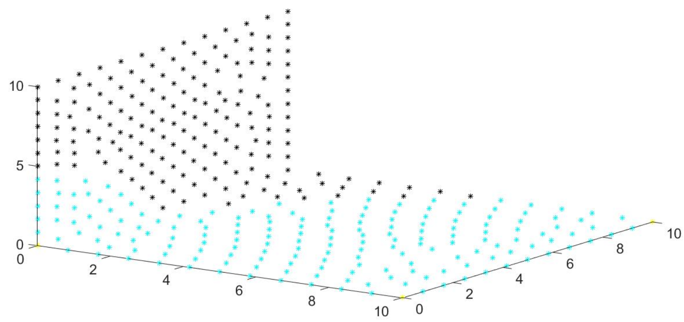


Figure 17: Group 2 of the cluster analysis EX1, displacements.

From a physical view, membrane displacements and shear displacements have different results. For this reason in this step, we are going only to study the degrees of freedom in displacements: shear displacements (perpendicular to each plate), membrane displacements with the same direction (X-axis direction in both plates) and membrane displacements with different directions (a plate with Y-axis direction and the other with Z-axis direction).

In the first case, shear displacements, the transfer matrix does not converge in blocks, see Fig. 18, but when we do the analysis cluster, we observe that there are two differentiated groups, Fig. 19. To get it, we have iterated until the power 190 of the transfer matrix. Thus, in this case we can identify perfectly the two subgroups, Fig. 20 and Fig. 21. Nevertheless, the contrast is very low, $6.8857 \cdot 10^{-16}$ and we can not separate the subsystems.

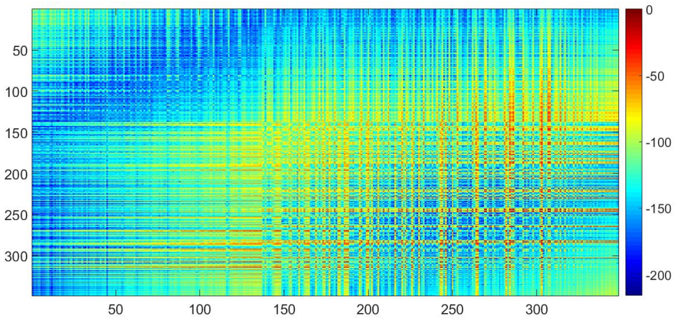


Figure 18: Colour plot of Example 1, case 1.

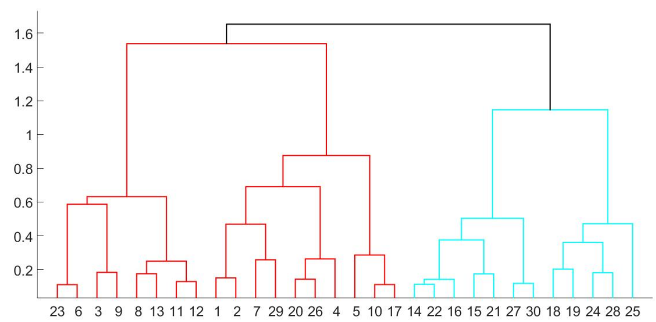


Figure 19: Dendrogram of Example 1, case 1.

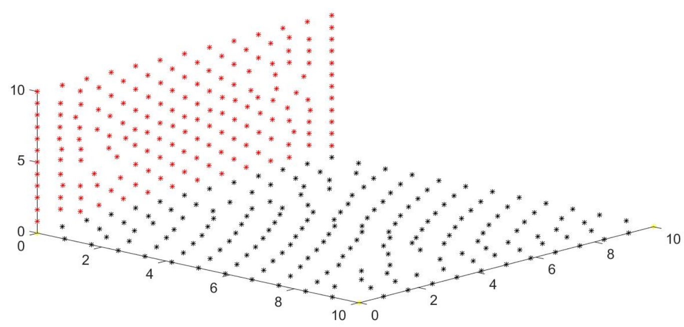


Figure 20: Group 1 of the cluster analysis EX1, case 1.

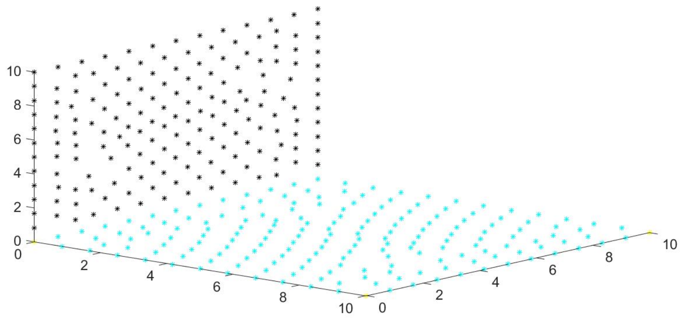


Figure 21: Group 2 of the cluster analysis EX1, case 1.

In the second case (membrane displacements with the same direction), we are not able to identify perfectly the subsystems, as we can see in the relation between the Dendrogram and its representation: Fig. 23, Fig. 24 and Fig. 25. It is noteworthy that the transfer matrix converges in a block matrix and it does for a power 40, less than the previous case. See Fig. 22.

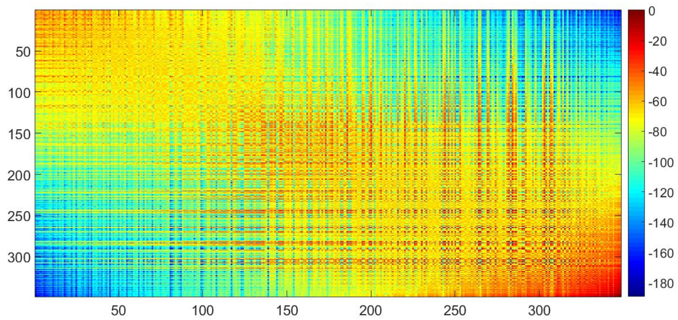


Figure 22: Colour plot of Example 1, case 2.

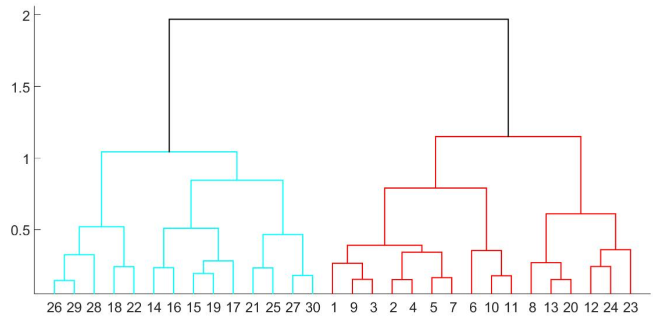


Figure 23: Dendrogram of Example 1, case 2.

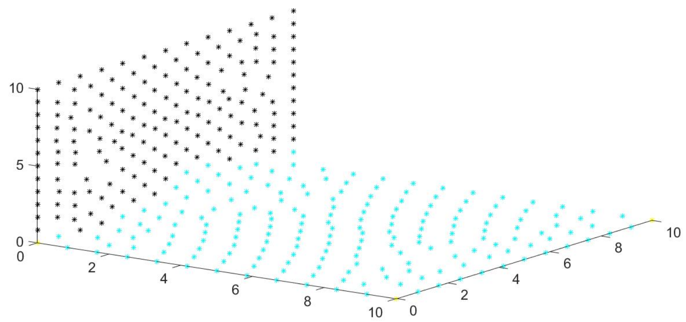


Figure 24: Group 1 of the cluster analysis EX1, case 2.

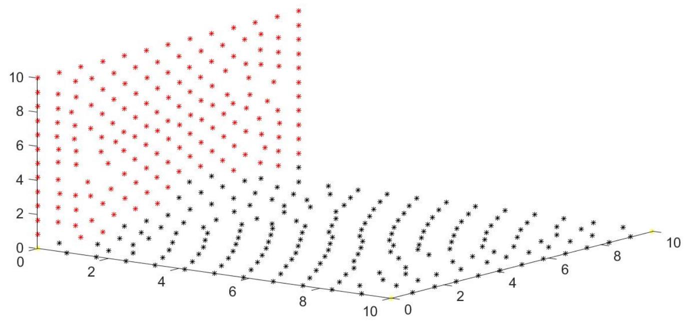


Figure 25: Group 2 of the cluster analysis EX1, case 2.

In both cases, the colour plot and cluster analysis have not the same results. Ultimately, the cluster analysis is the right one, because it is which really analyses the similarity. At the end, the colour plot is a simple graphical representation which gives us a first approximation.

In the third and last case, we can see that for a power 28 of the transfer matrix, it converges clearly in blocks, Fig. 26. At the same time, the cluster analysis makes a perfect identification of the subsystems, see the Dendrogram, Fig. 27, and the contrast has been increased to $8.0665{\cdot}10^{-4}$. In Fig. 28 and Fig. 29 we can see the representation of both subsystems.

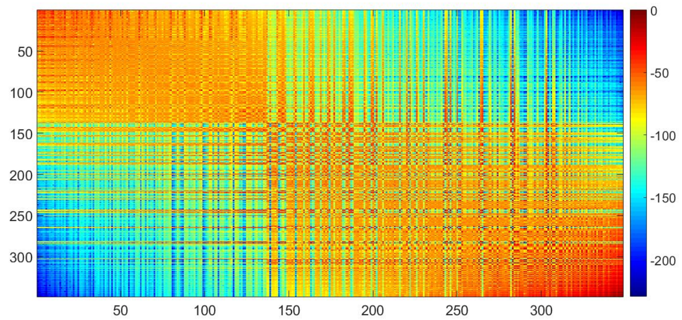


Figure 26: Colour plot of Example 1, case 3.

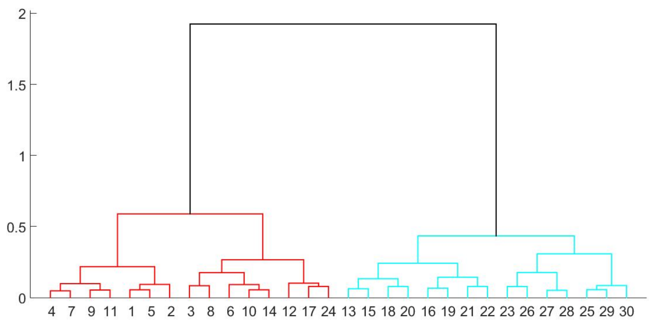


Figure 27: Dendrogram of Example 1, case 3.

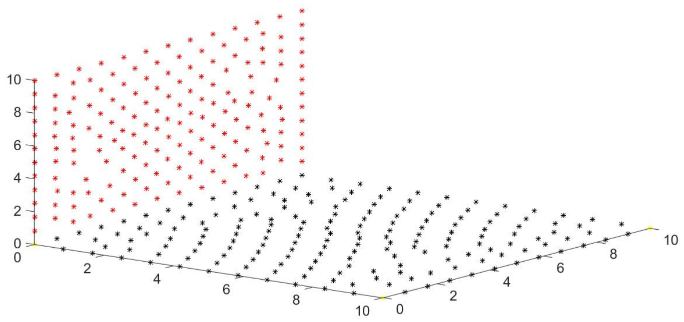


Figure 28: Group 1 of the cluster analysis EX1, case 3.

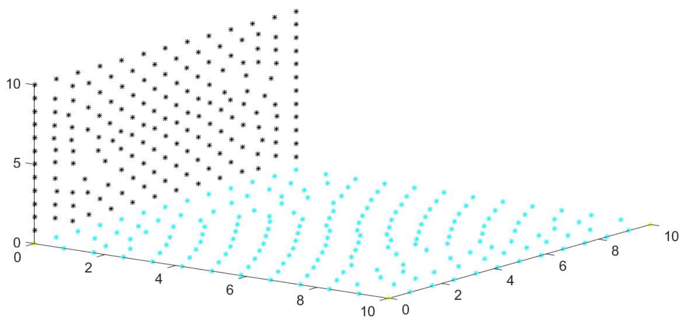
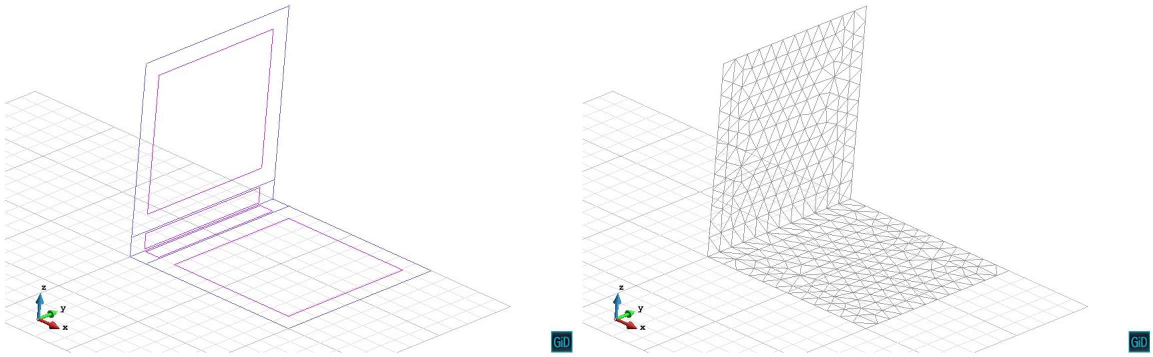


Figure 29: Group 2 of the cluster analysis EX1, case 3.

Finally, we can conclude in this first example, that a subdivision of the geometric domain gives us better results.

5.2 Example 2: Plates in 90° with aluminium joint

In this new example, we want to reduce a priori the coupling between the two plates. This will be achieved by modifying the properties of the connection, with an aluminium joint. In the next figures, we can see the geometry and its mesh, Fig. 30a and Fig. 30b. In the Table 2 there are the mechanical properties.



(a) Geometry. (b) Mesh.

Figure 30: Example 2, Plates and aluminium joint.

Element	Density(kg/m^3)	Poisson Ratio	Young Modulus(Pa)	Thickness(m)
Plates	7850	0.25	$206.9 \cdot 10^9$	0.025
Joint	2698	0.25	$70 \cdot 10^9$	0.025

Table 2: Mechanical properties of the Example 2.

In the same way as in the previous examples we see that it is preserved the good identification of subsystems, see the Colour Plot; Fig. 31, and the Dendrogram; Fig. 32. Due to modify the join properties, we check that the coupling is reduced, with a contrast equal to 0.0011. See Fig. 33 and Fig. 34 for to identify the subsystems graphically.

This result makes sense because we are reducing the stiffness of the joint, this causes that more energy will be absorbed in form of deformations, instead of it will be transmitted to the other subsystem.

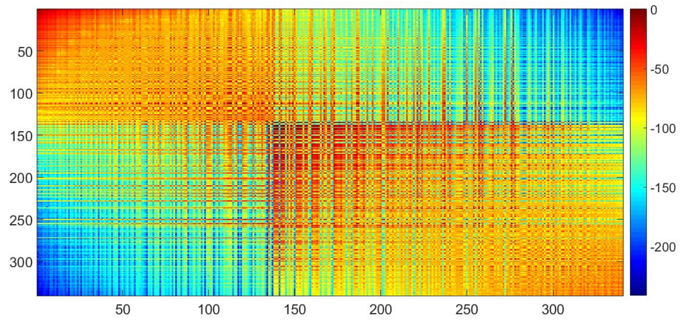


Figure 31: Colour plot of Example 2.

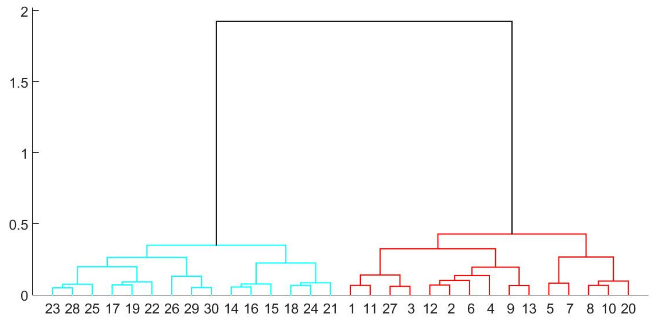


Figure 32: Dendrogram of Example 2.

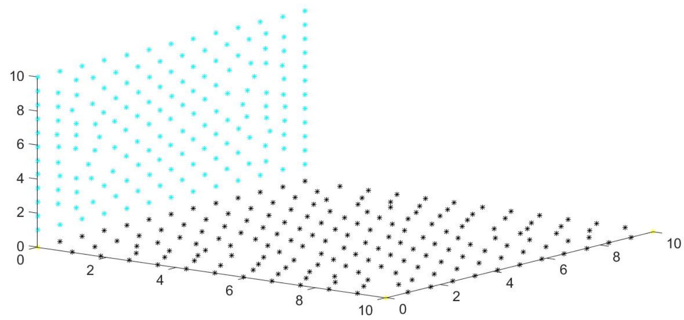


Figure 33: Group 1 of the cluster analysis EX2.

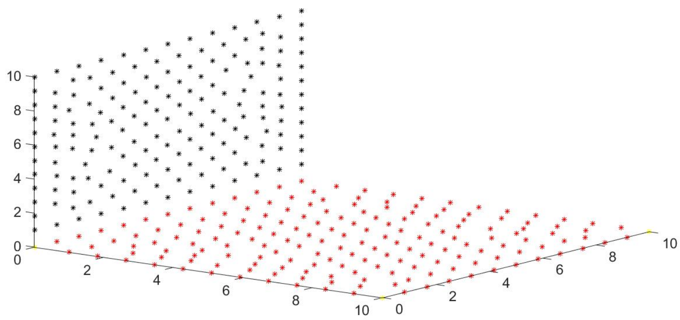
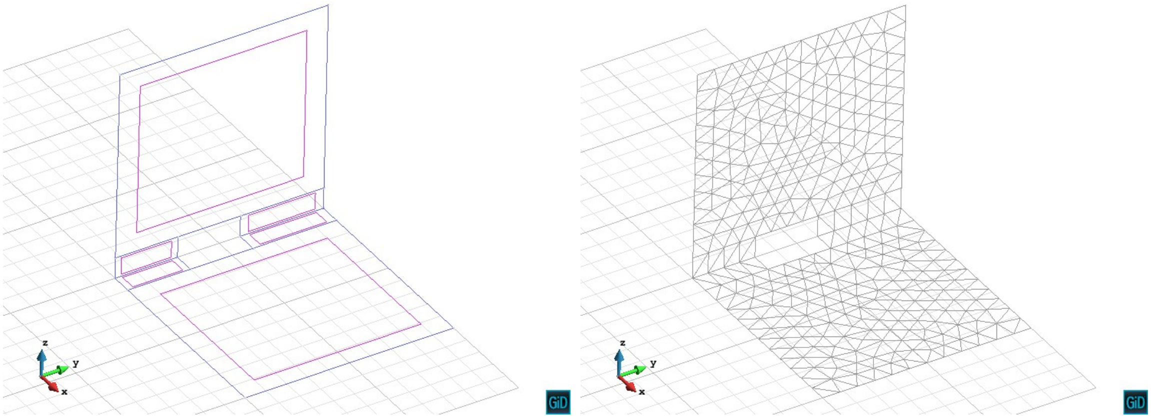


Figure 34: Group 2 of the cluster analysis EX2.

5.3 Example 3: Plates in 90º with soft joint

In the previous section we improved the identification of subsystems but the contrast still was low. In this new geometry, we want to try to increase the contrast as much as possible and identify perfectly the subsystems. This it will achieve creating gaps in the joint and modifying the mechanical properties of the plates and of the joint, in order to make a big difference. We are going to generate a factor 100 of difference, which could be compared with replacing the joint with a *revolute joint*.

In the following figures we can see the geometry and mesh generated, Fig. 35a and Fig. 35b. The chosen mechanical properties are given in the Table Fig. 3.



(a) Geometry. (b) Mesh.

Figure 35: Example 3, Plates in L with soft joint

Element	Density(kg/m^3)	Poisson Ratio	Young Modulus(Pa)	Thickness(m)
Plates	10000	0.25	$500{\cdot}10^9$	1
Joint	100	0.25	$5{\cdot}10^9$	0.01

Table 3: Mechanical properties of the Example 3.

In the last examples of L-plates, we have seen that if we do a very soft join, the transfer matrix will converge to a perfect block matrix, see Fig. 36, and in this case the power has been lower than in the previous ones, K = 25. This good convergence has resulted in a perfect identification of subsystems, as we can see in the Dendrogram, Fig. 37, and in the representation of these subgroups, Fig. 38 and Fig. 39.

As it could not be otherwise, the contrast has increased to 0.053.

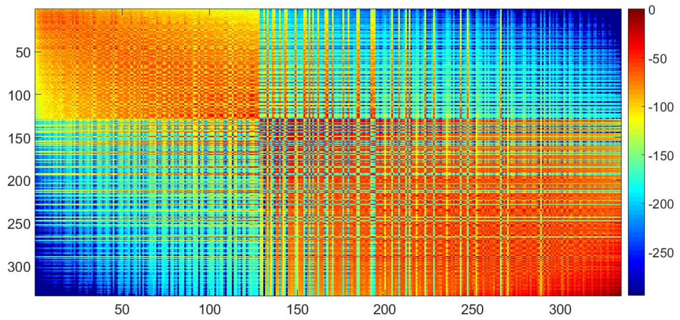


Figure 36: Colour plot of Example 3.

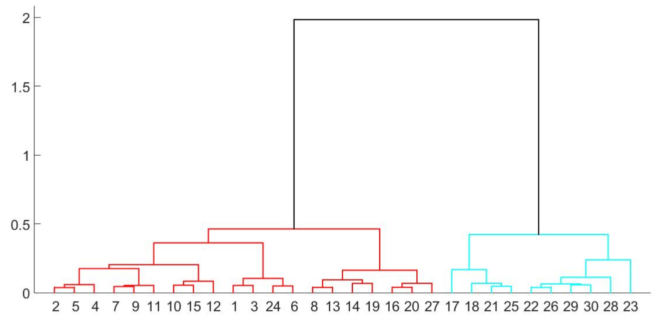


Figure 37: Dendrogram of Example 3.

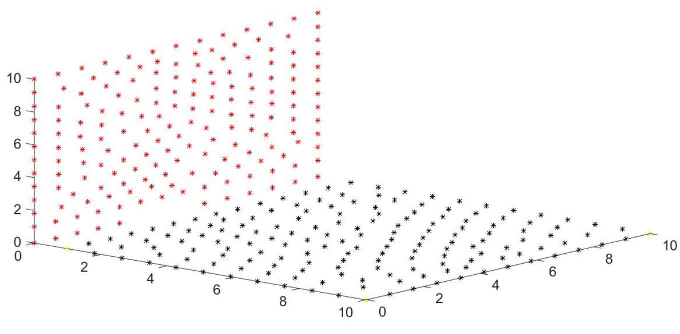


Figure 38: Group 1 of the cluster analysis EX3.

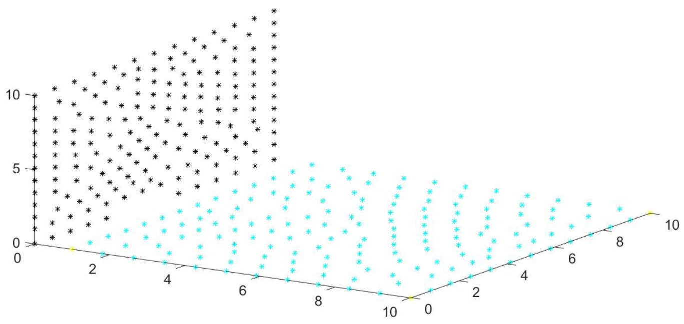


Figure 39: Group 2 of the cluster analysis EX3.

Now we can solve the dynamic problem using the Newmark method, introduced earlier, in order to check whether or not the subsystems can be analysed separately and to see how it affects the operational cost.

Thus, in the next figure, Fig.40, we can see the displacement on the X-axis of each node, where the blue line are the displacements calculating the whole system and the red line are the displacements calculating only the subsystem 1, isolated.

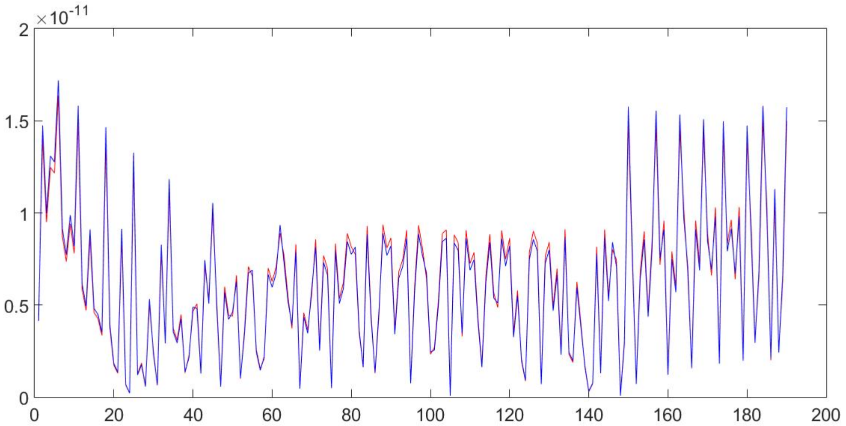


Figure 40: Displacements in X-axis of nodes from Subsystem 1.

The maximum error found is $8.1774{\cdot}10^{-13}$, far below the contrast, guaranteeing the condition in Eq.(14). In this case, we have reduce the system of equations from dimension 334x334 to 190x190.

6 Conclusions

The main conclusions of our thesis, after having a detailed study of technique of subsystems identification and analysing the results in representative examples evaluating its accuracy and applicability, through an automatic method designed in Matlab, are as follows:

1. The good subsystems identification from to observe the convergence in blocks of the powers of the Transfer matrix, through Colour Plot, it is only valid for cases where the blocks are very well defined. See Example 1, case 1, where the matrix does not converge, but a good identification of subsystems with other tools is performed.

2. Derived from the previous conclusion, Cluster Analysis is a good and reliable tool for automating the identification of subsystems. This analysis can be optimized, as we have seen in Example 1, isolating the degrees of freedom in displacements from the degrees of freedom in rotations, and even more, so if we separate the membrane displacements from the shear displacements. In other words, by applying a previous separation in domains, thanks to extra information of our object of study, we can optimize the analysis.

3. The subsystems identification depends mainly on two factors that we can find isolated or together:

   (a) The geometry of the structure, faced with the same stress, causes different reactions. This is linked to our study of degrees of freedom; the membrane displacements, of plates in different planes, will have different results. The larger difference between the results, the easier it will be to identify subsystems.

(b) Differences in the mechanical properties of the structure cause different behaviours at the same stress .

If we apply these two points to a structure, Example 3, the identification of subsystems is much more precise.

4. The degree of coupling between subsystems is analysed very well by the Contrast. We also observe that the lower power of the transfer matrix, when a favourable identification of subsystems has been made, the greater the contrast. That is, if we identify the subsystems very fast, it is very likely that they are weakly coupled systems. Whereas, if we need to greatly iterate the powers of the Transfer matrix, they are also likely to be strongly coupled subsystems.

7 References

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